

Mathematica 11.3 Integration Test Results

Test results for the 27 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x]^2)^3 dx$$

Optimal (type 3, 74 leaves, 4 steps):

$$a^3 x - \frac{b (3 a^2 - 3 a b + b^2) \operatorname{Coth}[c + d x]}{d} - \frac{(3 a - 2 b) b^2 \operatorname{Coth}[c + d x]^3}{3 d} - \frac{b^3 \operatorname{Coth}[c + d x]^5}{5 d}$$

Result (type 3, 266 leaves):

$$\begin{aligned} & - \frac{8 b^3 \operatorname{Cosh}[c + d x] (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]}{5 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} - \\ & \left(8 (15 a b^2 \operatorname{Cosh}[c + d x] - 4 b^3 \operatorname{Cosh}[c + d x]) (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^3 \right) / \\ & \left(15 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3 \right) - \\ & \left(8 (45 a^2 b \operatorname{Cosh}[c + d x] - 30 a b^2 \operatorname{Cosh}[c + d x] + 8 b^3 \operatorname{Cosh}[c + d x]) \right. \\ & \left. (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^5 \right) / \left(15 d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3 \right) + \\ & \frac{8 a^3 (c + d x) (a + b \operatorname{Csch}[c + d x]^2)^3 \operatorname{Sinh}[c + d x]^6}{d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^3} \end{aligned}$$

Problem 9: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csch}[c + d x]^2)^{5/2} dx$$

Optimal (type 3, 174 leaves, 8 steps):

$$\begin{aligned} & \frac{a^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}\right]}{d} - \frac{\sqrt{b} (15 a^2 - 10 a b + 3 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Coth}[c + d x]}{\sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}\right]}{8 d} - \\ & \frac{(7 a - 3 b) b \operatorname{Coth}[c + d x] \sqrt{a - b + b \operatorname{Coth}[c + d x]^2}}{8 d} - \frac{b \operatorname{Coth}[c + d x] (a - b + b \operatorname{Coth}[c + d x]^2)^{3/2}}{4 d} \end{aligned}$$

Result (type 3, 391 leaves):

$$\begin{aligned}
 & - \left(\left((-4 a^3 + 15 a^2 b - 10 a b^2 + 3 b^3) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{b} \operatorname{Cosh}[c + d x]}{\sqrt{-a + 2 b - a \operatorname{Cos} \left[2 \left(\frac{\pi}{2} - i (c + d x) \right) \right]}} \right] \right. \right. \\
 & \quad \left. \left. (a + b \operatorname{Csch}[c + d x]^2)^{5/2} \operatorname{Sinh}[c + d x]^5 \right) / \left(\sqrt{2} \sqrt{b} d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^{5/2} \right) + \right. \\
 & \quad \left((a + b \operatorname{Csch}[c + d x]^2)^{5/2} \left(-\frac{3}{2} (3 a b \operatorname{Cosh}[c + d x] - b^2 \operatorname{Cosh}[c + d x]) \operatorname{Csch}[c + d x]^2 - \right. \right. \\
 & \quad \left. \left. b^2 \operatorname{Coth}[c + d x] \operatorname{Csch}[c + d x]^3 \right) \operatorname{Sinh}[c + d x]^5 \right) / \left(d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^2 \right) + \\
 & \quad \left(4 a^3 (a + b \operatorname{Csch}[c + d x]^2)^{5/2} \left(-\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{b} \operatorname{Cosh}[c + d x]}{\sqrt{-a + 2 b + a \operatorname{Cosh}[2 (c + d x)]}} \right]}{\sqrt{2} \sqrt{b}} + \frac{1}{\sqrt{a}} \right. \right. \\
 & \quad \left. \left. \sqrt{2} \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[c + d x] + \sqrt{-a + 2 b + a \operatorname{Cosh}[2 (c + d x)]} \right] \right) \right. \\
 & \quad \left. \left. \operatorname{Sinh}[c + d x]^5 \right) / \left(d (-a + 2 b + a \operatorname{Cosh}[2 (c + d x)])^{5/2} \right) \right)
 \end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \operatorname{Csch}[c + d x]^2}} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a} \operatorname{Coth}[c + d x]}{\sqrt{a + b \operatorname{Csch}[c + d x]^2}} \right]}{\sqrt{a} d}$$

Result (type 3, 97 leaves):

$$\begin{aligned}
 & \left(\sqrt{-a + 2 b + a \operatorname{Cosh}[2 (c + d x)]} \operatorname{Csch}[c + d x] \right. \\
 & \quad \left. \operatorname{Log} \left[\sqrt{2} \sqrt{a} \operatorname{Cosh}[c + d x] + \sqrt{-a + 2 b + a \operatorname{Cosh}[2 (c + d x)]} \right] \right) / \\
 & \quad \left(\sqrt{2} \sqrt{a} d \sqrt{a + b \operatorname{Csch}[c + d x]^2} \right)
 \end{aligned}$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1 - \text{Csch}[x]^2} \, dx$$

Optimal (type 3, 26 leaves, 5 steps):

$$\text{ArcSin}\left[\frac{\text{Coth}[x]}{\sqrt{2}}\right] + \text{ArcTanh}\left[\frac{\text{Coth}[x]}{\sqrt{2 - \text{Coth}[x]^2}}\right]$$

Result (type 3, 65 leaves):

$$\frac{1}{\sqrt{-3 + \text{Cosh}[2x]}} \sqrt{2 - 2 \text{Csch}[x]^2} \left(\text{ArcTan}\left[\frac{\sqrt{2} \text{Cosh}[x]}{\sqrt{-3 + \text{Cosh}[2x]}}\right] + \text{Log}\left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]}\right] \right) \text{Sinh}[x]$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 - \text{Csch}[x]^2}} \, dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\text{Coth}[x]}{\sqrt{2 - \text{Coth}[x]^2}}\right]$$

Result (type 3, 45 leaves):

$$\frac{\sqrt{-3 + \text{Cosh}[2x]} \text{Csch}[x] \text{Log}\left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]}\right]}{\sqrt{2 - 2 \text{Csch}[x]^2}}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-1 + \text{Csch}[x]^2} \, dx$$

Optimal (type 3, 33 leaves, 6 steps):

$$-\text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-2 + \text{Coth}[x]^2}}\right] - \text{ArcTanh}\left[\frac{\text{Coth}[x]}{\sqrt{-2 + \text{Coth}[x]^2}}\right]$$

Result (type 3, 68 leaves):

$$\left(\sqrt{2} \sqrt{-1 + \text{Csch}[x]^2} \left(\text{ArcTan}\left[\frac{\sqrt{2} \text{Cosh}[x]}{\sqrt{-3 + \text{Cosh}[2x]}}\right] + \text{Log}\left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]}\right] \right) \text{Sinh}[x] \right) / \left(\sqrt{-3 + \text{Cosh}[2x]} \right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1 + \text{Csch}[x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

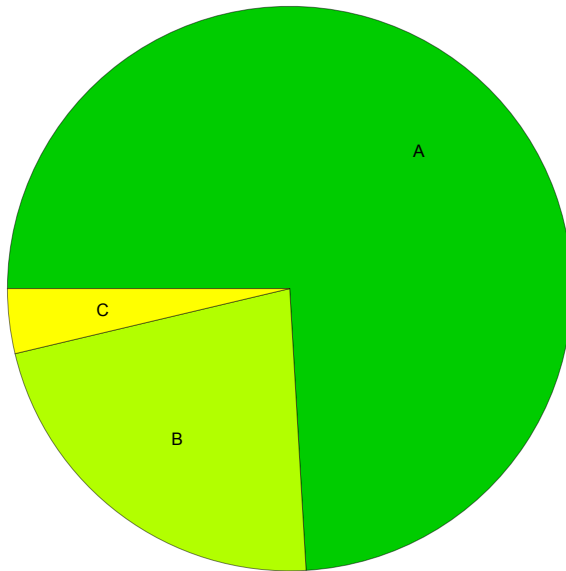
$$\text{ArcTan}\left[\frac{\text{Coth}[x]}{\sqrt{-2 + \text{Coth}[x]^2}}\right]$$

Result (type 3, 48 leaves):

$$\frac{\sqrt{-3 + \text{Cosh}[2x]} \text{Csch}[x] \text{Log}\left[\sqrt{2} \text{Cosh}[x] + \sqrt{-3 + \text{Cosh}[2x]}\right]}{\sqrt{2} \sqrt{-1 + \text{Csch}[x]^2}}$$

Summary of Integration Test Results

27 integration problems



A - 20 optimal antiderivatives

B - 6 more than twice size of optimal antiderivatives

C - 1 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts